Advances in Complex Systems
Vol. 17, No. 1 (2014) 1450003 (16 pages)

DOI: 10.1142/S0219525914500039

THE ROLE OF BILINGUALS IN LANGUAGE COMPETITION

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Received 29 July 2013
Revised 18 November 2013
Accepted 12 December 2013
Published 13 March 2014

We study the role played by bilinguals in the competition between two languages and in the formation of a bilingual community. To this aim we introduce a simple three-state model that combines the Minett–Wang model, in which the bilinguals do not affect directly the probability of transition of an individual from monolingualism to bilingualism, and the Baggs–Freedman model, in which such a transition probability only depends on bilinguals. The model predicts the possibility for the existence of a stable bilingual community though no particular conditions are assumed for the two competing languages: the asymptotic stability of the bilinguals community is only due to the type of dynamics regulating the transitions between different linguistic groups. The proposed model and the obtained results give some suggestions for the conditions necessary for the formation of a stable bilingual community. First of all, it is important that the bilinguals are valid representatives of the two languages, in the sense that they are regarded by monolinguals also as speakers of the other language. Besides that, the transition from bilinguals to monolinguals must be smaller than the opposite transition.

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Unless both these conditions are fulfilled, the stable equilibrium solution of the language competition is a monolingual society.

Keywords: Language competition; bilingualism; language dynamics; three-state model; language policies.

1. Introduction

Language shift is the process whereby a community of a language shifts to speaking another language. Typically, languages perceived to have a *higher status* spread at the expense of other languages that are considered by their own speakers to be *lower status* languages. In sociolinguistic studies of language shift, there are a few questions that have long remained unanswered and puzzle linguists. Namely, how can highly segregated languages, such as Picard in Northern France or Quechua in Peru still remain "alive" nowadays, while many other neighboring languages with analogous low status have disappeared long ago? Why has Breton historically disappeared first in towns, while other minority languages still thrive in urban settings — such as Picard, in the Lille–Roubaix–Tourcoing, whereas most rural varieties are endangered? Similarity between the dominant and the dominated languages has been assumed as an asset for minority language maintenance, fostering bilingualism instead of language shift [13–15]. Nevertheless, after nearly 30 years of cultural autonomy, it seems that Galician is relatively less successful than Basque in terms of language shift reversal, when taking into account the similarity of the former language with Spanish, and the challenge of learning the latter. What could be roughly dubbed ecological parameters, such as critical mass of demographic and social networks, prestige (or status) versus loyalty, geography (frontiers, ecological niches, insulation versus crossroad location), social settings (urban versus rural, open society versus small worlds or niches), socioeconomic payoff and volatility (social and psychosocial) make the difference for each situation [18].

However, none of these paradoxical situations turns out to be unpredictable in the light of complex system theory: they can all be accounted for by mathematical models, provided we use the proper set of ecological parameters, among those mentioned above. Moreover, the next horizon for modeling language shift and maintenance ecology will undoubtedly be the realm of globalized multilingualism, taking into account the whole range of repertoires, in native societies as well as in the diversified multilingual urban contexts of globalization. The gap between the huge diversity of concrete situations sociolinguists can observe on the spot, doing fieldwork and the state of the art of modeling language competition dynamics at abstract level, substantiates the need for models simulating clear-cut situations, i.e. canonical scenarios. Hence, how bilingual and multilingual settings fluctuate in space and time can be described or understood more easily on the basis of canonical patterns, as we attempted to do here.

Such canonical models have been provided during more than a decade by physicists, who have applied the tools of statistical mechanics and complex systems to study the problems that traditionally belong to the field of linguistics. The interest among physicists for modeling language competition was burst by the work of Abrams and Strogatz [1], see [22, 23]. However, the dynamical modeling of the interaction between linguistic communities had begun already more than a decade before with the works of Baggs and Freedman [3, 4].

In order to investigate competition between two languages, various two- and three-state models have been developed, see [18] for a review. In the first case, the system consists of two communities of monolingual speakers of two different languages, while in three-state models also bilinguals who speak both languages are also present.

The simple zero-dimensional (i.e., when no space variables are included) two- and three-state models, like the Abrams–Strogatz [1] or the Minett–Wang model that extends the Abrams–Strogatz work by including bilinguals [12], predict that whenever two languages compete for speakers, one language will eventually become extinct. Which of the two languages dies, depends on the initial proportions of speakers of each language and their relative status. Similar conclusions concerning language dominance or coexistence are reached also when considering the microscopic agent-based version of the Abrams and Strogatz model [20] or of the Minett–Wang model [6].

However, this holds only in the regime of small or neutral volatility, corresponding to values of the "volatility parameter" $a \geq 1$, see Sec. 2 for details. Volatility represents the tendency of an individual to change language. In the high volatility regime, the same models as well their microscopic versions can lead to opposite results, i.e., to language coexistence, under suitable conditions, see [21].

Specific underlying network topologies may be a crucial ingredient to obtain language coexistence as well. For example, they can generate metastable states with power law distributed life times, meaning in practice that coexistence can be observed on (arbitrary) long time scales, even if in principle such metastable do not live forever [5, 6].

There are other known mechanisms that can make a simple model system evolve toward stable coexistence of different languages in the low or neutral volatility regime. An instance is based on the similarity between the competing languages, considered in the model of Mira *et al.* [13–15]. Furthermore, a suitable form of population dynamics can generate stable bilingualism [3, 19]. Population dynamics is a necessary ingredient also for obtaining the coexistence of the two monolingual communities together with the associated bilinguals community in the model studied in [4].

Adding space dimensions to a model can transform its dynamics qualitatively. Spatial variables can be used to describe inhomogeneities due to the influence of a local culture or physical geography; they can lead to the survival of both languages, each mostly concentrated in different geographical areas [16, 17].

Surprisingly enough, the idea that mutual intelligibility (and consequently, structural similarity) might be an asset for the weaker competing language, giving it more chance to disappear less quickly, modeled in [13–15], is not much debated in current sociolinguistic literature. The reason is probably that we can also observe that throughout history, it can also happen the other way round — a mutually intelligible language Y gets scorned and despised because its similarity with the dominant language X allows to consider it as a dialect of the latter. This was particularly the case of the s.c. Oïl languages (or Oïl dialects) of Northern France. What we can positively say, however, is that in conditions of additive bilingualism with some political action in favor of a minority language, structural similarity helps because it makes it much easier to learn/acquire. If the status of language Y is on the rise, because of such an additive policy, representing an asset in social or professional life, its structural similarity makes it easier to learn, and therefore, more liable to spread again more in society.

In the present paper, we address the role played by bilinguals in the competition between two languages and in the formation of a bilingual community. To this aim we introduce a simple three-state model with a new form of transition probabilities between different linguistic groups, obtained by combining the Minett–Wang model [12], in which the bilinguals do not affect the probability of transition of an individual from monolingualism to bilingualism, and a simplified version of the Baggs–Freedman model [4], in which such transition is determined only by the bilinguals. The model predicts the possibility for the existence of a stable bilingual community though no particular conditions are assumed for the two competing languages, the model is zero-dimensional, and population dynamics is neglected. The latter simplifying hypotheses demonstrate that neither geography nor population dynamics represent necessary ingredients: in this case the asymptotic stability of the bilinguals community is only due to the type of dynamics regulating the transitions between different linguistic groups.

It is to be noticed that neglecting the terms of population dynamics does not mean neglecting population dynamics in general, possibly implying that the model is unrealistic in that respect, but only that the model validity is limited to situations of homogeneous growth. This point can be made more precise considering M linguistic groups with population sizes $N_m(t)$ $(m = 1, \ldots, M)$ at time t, described by the set of coupled differential equations

$$\frac{dN_m}{dt} = \sum_{n(n \neq m)} [F_{mn}(\mathbf{N}) - F_{nm}(\mathbf{N})] + rN_m \left(1 - \frac{\sum_n N_n}{K}\right), \tag{1}$$

where $F_{mn}(\mathbf{N})$ is the flux of individuals leaving the nth group to join the mth group, depending in general on the population sizes $\mathbf{N} = \{N_1, \ldots, N_M\}$, as in the models considered below. The parameters r and K are the common Malthus rate and carrying capacity, respectively; the presence of the total population size $N = \sum_n N_n$ in the numerator means that at any time t all available resources are accessible to all populations in the same way. From Eq. (1) one obtains that the total population evolves according to the Verhulst dynamics $dN/dt = rN(1 - N/K)$, whose asymptotic solution is $N(t\to\infty) = K$, leading to the asymptotic disappearance of all the population dynamics terms in Eq. (1). Therefore, the models considered here could describe the interaction between linguistic groups that have already reached a state in which reproduction and access to resources takes place in similar ways. This can be, e.g., the case of French and English in Canada, or Catalan and Castillan in Spain. However, the model cannot represent very asymmetrical situations with a minority language suffering a drastically reduced access to resources in comparison to the high-status language community, or a situation where linguistic groups do not share their own resources; the latter situation could be suitably described by the model of Kandler [11].

The model proposed in the present paper and the obtained results give some suggestions for the conditions necessary for the formation of a stable bilingual community. First of all, it is important that the bilinguals are considered as representatives of the two languages, i.e., that they do not form a separate third linguistic group, but may be regarded by a monolingual as members of the monolinguals community of the other language, so that they can influence the transitions from monolinguals of one language to bilinguals in an analogous way. Second, the transition from bilinguals to monolinguals has to be rare enough, as specified below. Unless both of these conditions are fulfilled, the stable equilibrium solution of the language competition is a monolingual society.

2. The Model of Minett and Wang: Survival of One Language Only

We start by recalling the model proposed in [12] by Minett and Wang. In that model, there are three types of speakers: people who speak only language X, people who speak only language Y and people who speak both languages X and Y; the latter type of speakers are denoted by Z. The proportions of speakers of X, Y and Z are

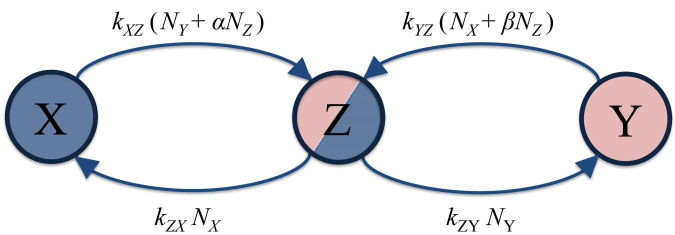

Fig. 1. (Color online) General scheme for the transition rates. In the Minett and Wang model, discussed in Sec. 2, $\alpha = \beta = 0$. In the model proposed in Sec. 3, $\alpha = \beta = 1$, corresponds to the situation when the bilinguals are considered by monolinguals of one language as representatives of the other language. In the case discussed in Sec. 4, α and β can assume arbitrary values in the range $[0, 1]$.

N_X, N_Y and N_Z, respectively. It is assumed that only transitions $X \to Z$, $Z \to X$, $Y \to Z$ and $Z \to Y$ occur, i.e., the speakers of X do not become the speakers of Y directly, and vice versa, but it is possible only through the state Z (see Fig. 1). In the original Minett–Wang model, the rates of change of the monolingual population sizes are

$$\begin{aligned} \frac{dN_X}{dt} &= k_{ZX} N_X^a N_Z - k_{XZ} N_Y^a N_X, \\ \frac{dN_Y}{dt} &= k_{ZY} N_Y^a N_Z - k_{YZ} N_X^a N_Y, \end{aligned} \tag{2}$$

describing how $N_X(N_Y)$ grows due to the bilinguals becoming monolinguals of $X(Y)$ and decreases due to the monolinguals of $X(Y)$ becoming bilinguals. From Eqs. (2) one can see that the transition rates of an individual from the population i of the initial state to population f of the final state ($i, f = X, Y, Z$) are proportional to (a) a suitable effective rate constant k_{if}; (b) a power a, usually referred to as "volatility parameter" [21], of the size of the corresponding "attracting population"; and (c) to the size of the origin population N_i. The rate constants k_{XZ}, k_{YZ}, k_{ZX} and k_{ZY} describe the interplay of various factors and can be interpreted in many ways: for example in [12] each of these parameters is expressed in turn through other three parameters taking into account the mortality rate, language status and other sociolinguistic factors. While maintaining a general operative approach assuming that the k's are effective parameters of the model, describing e.g., time scales and possible asymmetries between the two competing languages, we remark that their larger or smaller numerical value may be due to a number of factors such as the higher or lower status of the corresponding language or other proper (known or unknown) qualitative aspects.

By "attracting population" it is meant here that part of the total population that influences the transition probability of an individual. By inspection of Eqs. (2) one can see that the attracting population for a monolingual undergoing a transition to the bilingual state is the monolinguals population of the other language. Also for the opposite transition from bilingual to monolingual, the attracting population is represented by the monolinguals population of the final state.

Finally, the volatility parameter a describes the tendency to switch to another linguistic group [21]. A high volatility regime (with frequent language changes) producing at equilibrium language coexistence is obtained for values smaller than a suitable critical value, $a < a_{\rm cr}$, while for values $a > a_{\rm cr}$ the low volatility regime produces language dominance [7, 21].

In the present work, we study the role of bilinguals in language competition by analyzing different functional forms of the attracting population. As shown below, a novel mechanism is found, that can lead the system to stable bilingualism even in volatility regimes where other models predict language dominance; thus, for the sake of clarity in the following we only consider the neutral volatility case $a = 1$ (giving Lotka–Volterra-type models), in which transition probabilities are proportional to the size of the attracting populations. We start from the version of the Minett–Wang model obtained for $a = 1$, defined by the two following equations,

$$\begin{aligned} \frac{dN_X}{dt} &= k_{ZX} N_X N_Z - k_{XZ} N_Y N_X, \\ \frac{dN_Y}{dt} &= k_{ZY} N_Y N_Z - k_{YZ} N_X N_Y. \end{aligned} \tag{3}$$

Here the rates of change are simply proportional both to the origin population and to the attracting population. Because $N_X + N_Y + N_Z = 1$ then $dN_Z/dt = -dN_X/dt - dN_Y/dt$. Replacing in Eqs. (3) $N_Z = 1 - N_X - N_Y$ one can write,

$$\begin{aligned} \frac{dN_X}{dt} &= N_X[k_{ZX}(1 - N_X - N_Y) - k_{XZ} N_Y], \\ \frac{dN_Y}{dt} &= N_Y[k_{ZY}(1 - N_X - N_Y) - k_{YZ} N_X]. \end{aligned} \tag{4}$$

The dynamics of the model described can be analyzed by seeking the equilibrium values of N_X, N_Y, N_Z for which the rates of change are zero and by studying the direction field. The model has four equilibrium points: $(1, 0, 0)$ corresponding to all individuals being monolinguals in X; $(0, 1, 0)$ corresponding to all individuals being monolinguals in Y; $(0, 0, 1)$ corresponding to all individuals being bilinguals; $(k_{XZ}k_{ZY}/\Sigma, k_{ZX}k_{YZ}/\Sigma, k_{XZ}k_{YZ}/\Sigma)$, with $\Sigma = k_{XZ}k_{ZY} + k_{ZX}k_{YZ} + k_{XZ}k_{YZ}$, corresponding to a state in which there are speakers of each type. Using linear stability analysis, one can show that the states in which everybody is bilingual or in which there are speakers of each type, are unstable for all parameter values, meaning that even if these states are approached, the system will subsequently tend to move away from them. Instead, the first two equilibrium points are stable, meaning that once the system has approached either of these two states, it will remain there. Therefore, the model discussed always leads to the situation when in the end all the individuals will speak the same language, i.e., one of the competing languages wins and the other one disappears. This can be seen also from the direction field, presented in Fig. 2(a) for fixed parameter values $k_{XZ} = 1$, $k_{YZ} = 0.3$, $k_{ZX} = 0.1$ and $k_{ZY} = 0.2$ (notice that we have rescaled here all the parameters k by the largest of them so that the largest value is 1 and the other values are in the range $(0, 1)$), representing a situation where language Y has a higher status than language X and it is more common that monolinguals become bilinguals than the opposite transition. The two solid lines represent the trajectories in the $N_X - N_Y$ plane of two possible situations in which language X or Y wins, depending on the initial proportions of speakers — it is assumed that initially there are only monolinguals of languages X and Y and no bilinguals, corresponding to trajectories starting from a point located along the diagonal (dotted line), defined by $N_X + N_Y = 1$. Varying the parameters k changes the position of the fourth equilibrium point and the value of the critical initial fraction of speakers leading to the extinction of one or the other language, but not the general dynamics of the system.

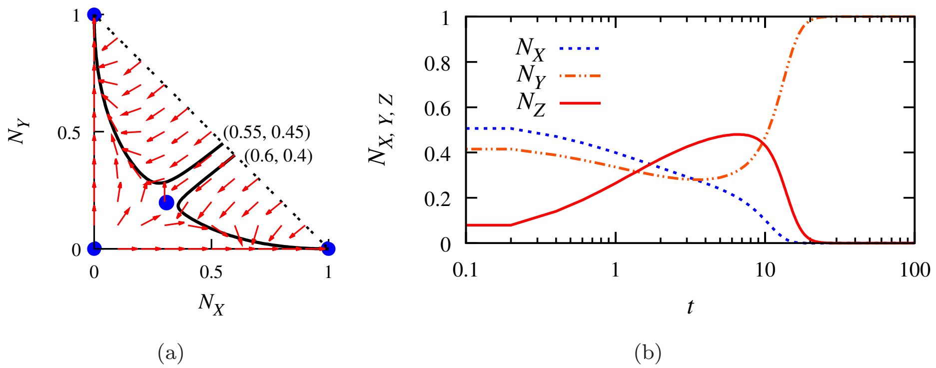

Fig. 2. Model of Minett and Wang described by Eqs. (3), or equivalently by Eqs. (4), with parameters $k_{ZX} = 0.4$, $k_{ZY} = 0.5$, $k_{YZ} = 0.8$, $k_{XZ} = 1$. (a) Direction field. The solid black lines correspond to two trajectories with different initial conditions leading to different stable equilibrium solutions: if at $t = 0$ $N_X < 0.586$ ($N_Z = 0$), it is language Y with the higher status that wins; if $N_X > 0.586$ ($N_Z = 0$) language X wins despite its lower status. (b) Time evolution of the same system with initial condition $N_X(t = 0) = 0.55$, $N_Y(t = 0) = 0.45$, $N_Z(t = 0) = 0$.

It is interesting to investigate also the time evolution of the system. Figure 2(b) presents the time evolution of the population fractions corresponding to the upper trajectory in Fig. 2(a), with an initial larger fraction of speakers of language X with a lower status, $N_X = 0.55$, and a smaller fraction of speakers of language Y with a higher status, $N_Y = 0.45$. Despite the initially smaller number of speakers, it is language Y that eventually prevails and language X gets extinct. For the given parameter values we also observe that for some time the system consists mostly of bilinguals — for some parameter values this time period can be made very long. However, when the monolingual community of language X disappears, also the bilingual community disappears very fast, since in this case $dN_Z/dt = -dN_Y/dt = -k_{ZY}N_YN_Z$. Instead, if $N_X(t = 0) > 0.586$ language X wins the competition despite its lower status, as in the situation represented by the lower trajectory in Fig. 2(a) (time evolution not shown).

3. Bilinguals as Representatives of the Other Language: Stable Bilingual Community

In the model of Minett and Wang, discussed in Sec. 2, it is assumed that the rate of change of a monolingual of $X(Y)$ to become a bilingual is proportional to $N_Y(N_X)$, i.e., the decision to learn the other language is not influenced by the total number of its speakers, but only by the number of monolingual speakers of that language. Here, instead, we assume that the rate of change of a monolingual of $X(Y)$ to become a bilingual is proportional to the total number of speakers of language $Y(X)$ including bilinguals, i.e., to the sum $N_Y + N_Z$ ($N_X + N_Z$) (see Fig. 1). Then the system is described by the following equations:

$$\begin{aligned}\frac{dN_X}{dt} &= k_{ZX}N_XN_Z - k_{XZ}(N_Y+N_Z)N_X,\\ \frac{dN_Y}{dt} &= k_{ZY}N_YN_Z - k_{YZ}(N_X+N_Z)N_Y,\end{aligned} \tag{5}$$

which, replacing $N_Z = 1 - N_X - N_Y$, become,

$$\begin{aligned}\frac{dN_X}{dt} &= N_X[k_{ZX}(1-N_X-N_Y) - k_{XZ}(1-N_X)],\\ \frac{dN_Y}{dt} &= N_Y[k_{ZY}(1-N_X-N_Y) - k_{YZ}(1-N_Y)].\end{aligned} \tag{6}$$

Similarly to the model of Minett and Wang, this model has four equilibrium points. Also in this case the fourth equilibrium point, $(k_{XZ}(k_{ZY} - k_{YZ})/\sigma, k_{YZ}(k_{ZX} - k_{XZ})/\sigma, k_{XZ}k_{YZ}/\sigma)$, with $\sigma = k_{XZ}k_{ZY} + k_{ZX}k_{YZ} - k_{XZ}k_{YZ}$, corresponding to a state in which there are speakers of each type, is unstable for all parameter values (notice that this equilibrium point exists only if $k_{ZX} > k_{XZ}$ and $k_{ZY} > k_{YZ}$). The stability of the other three equilibrium points depends on the parameter values. The eigenvalues corresponding to $(1, 0, 0)$ are $\lambda_1 = -k_{YZ}$, $\lambda_2 = k_{XZ} - k_{ZX}$, to $(0, 1, 0)$ $\lambda_1 = -k_{XZ}$, $\lambda_2 = k_{YZ} - k_{ZY}$ and to $(0, 0, 1)$ $\lambda_1 = k_{ZX} - k_{XZ}$, $\lambda_2 = k_{ZY} - k_{YZ}$. This means that if the parameters k_{XZ} and k_{YZ} corresponding to monolinguals of X and Y becoming bilinguals are larger than the parameters k_{ZX} and k_{ZY} corresponding to bilinguals becoming monolinguals of X and Y (i.e., $k_{XZ} > k_{ZX}$ and $k_{YZ} > k_{ZY}$) then the final state of the system is a stable bilingual community, independent of the language statuses and initial fractions of the speakers of the two languages, see the example of direction field in Fig. 3(a). If, instead, $k_{ZX} > k_{XZ}$ and $k_{ZY} > k_{YZ}$ then, depending on the initial conditions for N_X and N_Y, one or the other language will win the competition for the given parameter values [compare Fig. 3(b)]. The condition $k_{ZX} > k_{XZ}$ together with $k_{YZ} > k_{ZY}$ leads to the monolingual community of language X [Fig. 3(c)] and the condition $k_{ZY} > k_{YZ}$ together with $k_{XZ} > k_{ZX}$ to the monolingual community of language Y [Fig. 3(d)], independently of the language statuses and initial fractions of the speakers of the two languages.

The form of Eqs. (5), or equivalently of (6), defining the proposed model and the results obtained provide some suggestions for formulating the conditions necessary for the formation of a stable bilingual community in a society where initially two languages are spoken. First of all, it is important that the bilinguals are considered as actual representatives of the two languages on the same footing as monolinguals: that is, they should be able to influence the transition from monolinguals X (Y) to bilinguals analogously to the monolinguals Y (X). Besides that, the transitions from bilinguals to monolinguals should take place more rarely than the opposite transitions, i.e., in terms of parameters, $k_{XZ} > k_{ZX}$ and $k_{YZ} > k_{ZY}$. While both these conditions are rather natural in societies where two languages are used and bilingualism is appreciated, in many situations the opposite occurs. As one can see

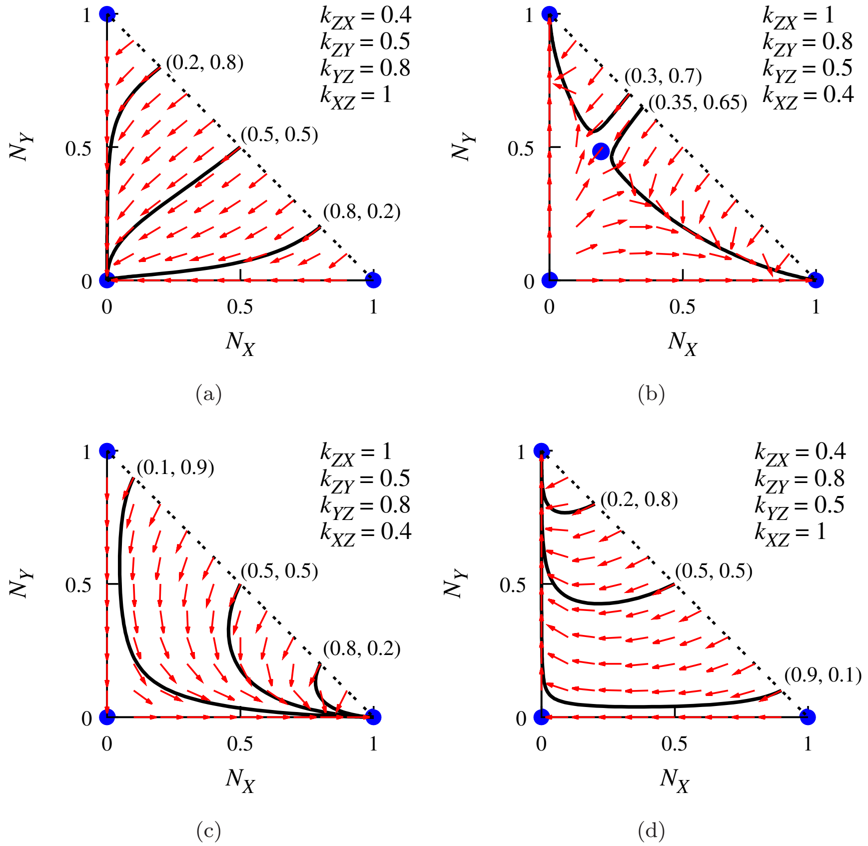

Fig. 3. Direction fields corresponding to four sets of parameters leading to different final scenarios for the model described by Eqs. (5), or equivalently by Eqs. (6), in which bilinguals are considered by the monolinguals of one language as representatives of the other language. (a) $k_{XZ} > k_{ZX}$ and $k_{YZ} > k_{ZY}$: the final state is a stable bilingual community, independent of the language statuses and initial fractions of X and Y speakers; (b) $k_{ZX} > k_{XZ}$ and $k_{ZY} > k_{YZ}$: depending on the initial conditions for N_X and N_Y one of the two languages wins the competition; (c) $k_{ZX} > k_{XZ}$ and $k_{YZ} > k_{ZY}$: the final state is the monolingual community of language X, independent of the language statuses and initial fractions of the speakers; (d) $k_{ZY} > k_{YZ}$ and $k_{XZ} > k_{ZX}$: the final state is the monolingual community of language Y, independent of the language statuses and initial fractions of the speakers.

from the examples relative to the Minett–Wang model in Fig. 2, where bilinguals are *not* considered as representatives of one or the other language, one of the languages will get extinct very fast, as soon as there are no more monolingual speakers of that language, even if at some point the system consists mostly of bilinguals. Instead, from Fig. 4 where the same values of the constants k's used in Fig. 2(b) have been employed, one can see that in the case of the model defined by Eqs. (5) the fraction of bilinguals continues to grow even after the extinction of the monolinguals of one of the languages, so that in the end both languages survive in a bilingual society.

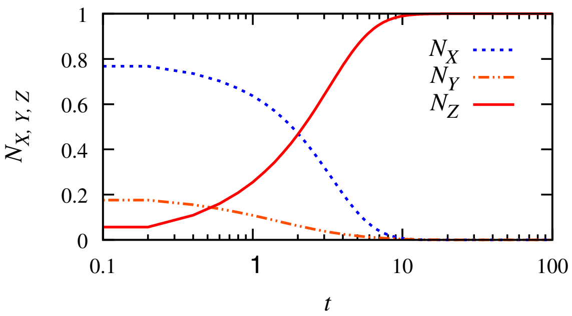

Fig. 4. Time evolution of the same system as in Fig. 3(a) with initial condition $N_X(t=0)=0.8$, $N_Y(t=0)=0.2$ and $N_Z(t=0)=0$.

4. The Influence of the Bilingualism Policy

In this section, the model introduced in the previous section is generalized with a two-fold aim. First, one may be interested in simulating a society where the monolinguals' consideration for the bilinguals community is intermediate between the Minett–Wang model, where bilinguals play no role in enhancing the transition from monolinguals to bilinguals, and the new model described in Sec. 3, where the monolinguals of one language and the bilinguals are considered on the same footing by the monolinguals of the other language. Furthermore, one may need a quantitative description of the effect of language policies.

When two competing languages have sufficiently different statuses, i.e., one of them is much more attractive then the other one, then the bilinguals community consists typically of the bilinguals coming from the lower status language group and the existence of the bilingual community may not enhance the attractiveness to learn the lower status language by the speakers of the higher status language. In this situation it is important to take some policy to support the tendency to do it. The situation may be modeled by the following equations (see Fig. 1):

$$\begin{aligned}\frac{dN_X}{dt} &= k_{ZX}N_XN_Z - k_{XZ}(N_Y+\alpha N_Z)N_X,\\ \frac{dN_Y}{dt} &= k_{ZY}N_YN_Z - k_{YZ}(N_X+\beta N_Z)N_Y,\end{aligned} \tag{7}$$

where the parameters $\alpha, \beta \in [0,1]$, may be interpreted as the importance of the bilinguals as the representatives of language Y, X to the monolinguals of X, Y, respectively. For $\alpha=\beta=0$ the model of Minett and Wang is recovered and for $\alpha=\beta=1$ we recover the model presented in Sec. 3. The model has four equilibrium points: $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and $(k_{XZ}(k_{ZY}-\beta k_{YZ})/\sigma_{\alpha\beta}, k_{YZ}(k_{ZX} - \alpha k_{XZ})/\sigma_{\alpha\beta}, k_{XZ}k_{YZ}/\sigma_{\alpha\beta})$, with $\sigma_{\alpha\beta} = k_{XZ}(k_{ZY}-\beta k_{YZ}) + k_{YZ}[k_{ZX}+k_{XZ}(1-\alpha)]$; the fourth equilibrium point exists only if $k_{ZY} > \beta k_{YZ}$ and $k_{ZX} > \alpha k_{XZ}$. The eigenvalues corresponding to the equilibrium point $(1,0,0)$ are $\lambda_1 = \alpha k_{XZ} - k_{ZX}$, $\lambda_2 = -k_{YZ}$, to $(0,1,0)$ $\lambda_1 = -k_{XZ}$, $\lambda_2 = \beta k_{YZ} - k_{ZY}$ and to $(0,0,1)$ $\lambda_1 =$

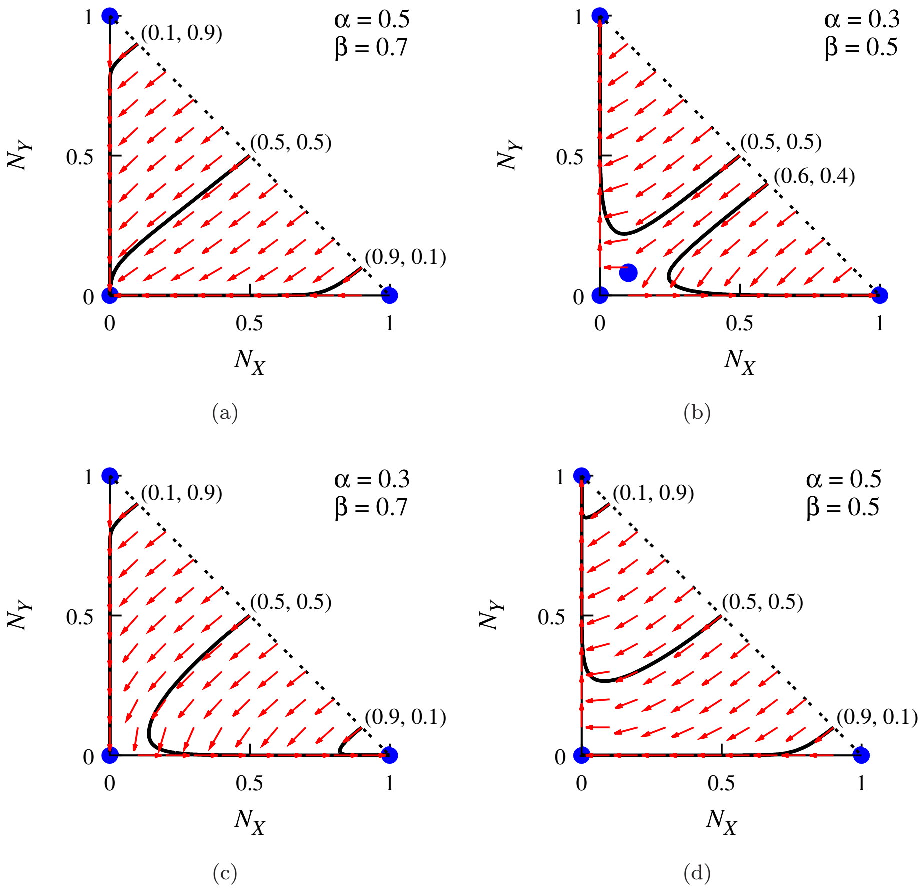

Fig. 5. The model described by Eqs. (7). The direction fields for the same values of k as in Figs. 2 and 3(a) and for different values of α and β leading to different scenarios ($\alpha_{\rm cr} = 0.4$ and $\beta_{\rm cr} = 0.625$): (a) if $\alpha > \alpha_{\rm cr}$ and $\beta > \beta_{\rm cr}$ then the final state of the system is a stable bilingual community, independently of the language statuses and initial proportions of the speakers of the two languages; (b) if $\alpha < \alpha_{\rm cr}$ and $\beta < \beta_{\rm cr}$ then, depending on the initial conditions for N_X and N_Y, one or the other language will win the competition; (c) if $\alpha < \alpha_{\rm cr}$ and $\beta > \beta_{\rm cr}$ then the final state is the monolingual community of language X, independently of the language statuses and initial fractions of the speakers of the two languages; (d) if $\alpha > \alpha_{\rm cr}$ and $\beta < \beta_{\rm cr}$ then the final state is the monolingual community of language Y, independently of the language statuses and initial fractions of the speakers of the two languages.

$k_{ZX} - \alpha k_{XZ}$, $\lambda_2 = k_{ZY} - \beta k_{YZ}$. From here we see that there exist critical values $\alpha_{\rm cr} = k_{ZX}/k_{XZ}$ and $\beta_{\rm cr} = k_{ZY}/k_{YZ}$, leading to the stability of one or the other equilibrium point. If $\alpha > \alpha_{\rm cr}$ and $\beta > \beta_{\rm cr}$ then the final state of the system is a stable bilingual community, independently of the language statuses and initial proportions of the speakers of the two languages [Fig. 5(a)]. The condition $\alpha, \beta \in [0, 1]$, in turn, implies that $k_{XZ} > k_{ZX}$ and $k_{YZ} > k_{ZY}$ for the possibility of the stable bilingual community. Instead, if $\alpha < \alpha_{\rm cr}$ and $\beta < \beta_{\rm cr}$ then, depending on the initial conditions for N_X and N_Y, one or the other language will win the competition [Fig. 5(b)], i.e., we basically recover the model of Minett and Wang with the only difference that the values of α and β influence the critical initial values of N_X and N_Y leading to the extinction of one or the other language. The condition $\alpha < \alpha_{\rm cr}$ together with $\beta > \beta_{\rm cr}$ leads to the monolingual community of language X [Fig. 5(c)] and the condition $\alpha > \alpha_{\rm cr}$ together with $\beta < \beta_{\rm cr}$ to the monolingual community of language Y [Fig. 5(d)], independently of the language statuses and initial fractions of the speakers of the two languages. Notice also that the condition $\beta > \beta_{\rm cr}$ implies that $k_{YZ} > k_{ZY}$ is necessary for the winning of language X, and the condition $\alpha > \alpha_{\rm cr}$ implies that $k_{XZ} > k_{ZX}$ has to be fulfilled for the winning of language Y (due to the condition $\alpha, \beta \in [0, 1]$). The fourth equilibrium point corresponding to a state in which there are speakers of each type, is unstable for all parameter values. The different situations are depicted in Fig. 5 for the same values of the parameters k as in Figs. 2 and 3(a), i.e., the parameters satisfy the conditions $k_{XZ} \geq k_{ZX}$ and $k_{YZ} \geq k_{ZY}$. For the given parameters, the critical values of α and β are $\alpha_{\rm cr} = 0.4$ and $\beta_{\rm cr} = 0.625$.

Though in a real situation it is difficult to measure the parameters α, β quantifying the importance of the bilinguals as the representatives of language Y, X to the monolinguals of X, Y, respectively, the conclusion of the results obtained is that the bilingualism policy — expected to increase the values of α and β — has an important impact on the language survival. Namely, for fixed values of k characterizing the transitions from monolinguals to bilinguals and from bilinguals to monolinguals, there exist critical values of α and β below which one of the languages gets extinct. The critical value of α (β) is the larger the lower is the status of language Y (X), i.e., the lower the status of a language is, the more support and attention it needs for surviving. However, at the same time one should not ignore the other language, which would otherwise get endangered. The best situation is obtained when both languages are given the same maximal importance through considering also its bilingual speakers as monolinguals so that people can freely choose in which language they prefer to speak.

5. Conclusion

We have studied a model of two interacting language communities and the corresponding bilingual community. In this model, two parameters α and β quantify the influence that the bilingual community can have in affecting the choice of a monolingual speaker in learning the other language, thus becoming bilingual. Depending on the values of α and β, as well as on the other system parameters, the stable solutions can represent either a society which is monolingual in one of the two languages or a society of bilinguals only. Bilingualism emerges here as a stable solution from the dynamics of the model only and represents an additional novel mechanism leading to language coexistence (in the form of bilingualism) besides others already known, mentioned in the Introduction, such as specific underlying network topologies, geographical or cultural inhomogeneities, or a high volatility regime as defined in [21].

The model presented can be considered as a generalization and a combination of the Minett–Wang model [12], in which the bilingual community does not affect the probability of transition of an individual from monolingual to bilingual, and of the Baggs–Freedman model [4], in which only the bilingual community (but not the monolingual community of the other language) directly affects the transition from monolingualism to bilingualism. By choosing different values of the parameters α and β, one can describe a wide set of different linguistic situations. Importantly, specific linguistic policies aimed at defending and revitalizing minority languages can be described by a change in the values of α and β, possibly turning a system evolution toward a monolingual community into one toward a bilinguals community.

It is to be noticed that the model introduced here better describes a symmetrical competition between two languages for various reasons: (a) language dynamics is formulated in symmetrical terms respect to the two languages; (b) the model variable N_Z, representing the fraction of bilinguals Z, does not distinguish between speakers X who also learned Y and speakers Y who also learned X, which is a relevant difference in real situations; (c) as discussed in the Introduction the population dynamics implicitly contained in the model is symmetrical both in its laws and parameter values. The important source of asymmetry here, between the two languages, affect (apart from the initial conditions) the numerical values of the transition constants k's.

Concerning the possible developments of the model, there remains the wide part of parameter space with volatility parameter $a \neq 1$ to be explored. It is known that in this part of the parameter space fragmentation and language coexistence can exist for a smaller than a critical value, $a < a_{\mathrm{cr}}$ [21], so that the natural intriguing question arises, how the new parameters α and β affect the shape of the region of language coexistence/dominance. Furthermore, special care should be devoted to reanalyze the case $a = 1$, since in some models it is critical, in that it divides the language dominance from the language coexistence domain of the parameter space.

Forthcoming studies should also take into account the distinction among the various types of bilingualism [9].

Another possible objection is that neither a purely bilingual nor a purely monolingual community is observed in real situations. In particular, the hypothesis that bilingualism always leads eventually to monolingualism (as originally advocated in the seventies in the works by Aracil [2]) does not fit the current reality. Instead, nowadays, e.g., in the study of the interaction between Castillan and the Catalan language, the paradigm of sociolinguistics is strongly committed to an interactionist and integrative approach of social networks [8], consistent with the fact that bilingualism may be widely observed in most of the territory where a co-official or protected language is competing with an official language, as in Catalunya or some sub-regions of the Gipuzkoa within the Basque Autonomous Community. However, at the same time, as far as we can observe bilingual situations, also stable bilateral bilingualism is still an abstract/ideal situation. Though there are many situations where bilingualism occurs, they cannot be considered stable. For example, Quebec bilingualism is far from stable — it is still very conflictual — and Switzerland is far more a confederation of bilingualisms and monolingualisms than a stable multilingual community. The situation is even more complicated in Belgium, where bilingualism might now be even decreasing, out of Flemish polarization against the French-speaking component of this supposedly bilingual community. More complex situations can be described invoking one of the mentioned mechanism, e.g., justifying the existence of a high volatility regime or taking into account the inhomogeneities that can be described at some level in the parameter space, e.g., geographical or cultural barriers [16,17], space modulations of a language prestige [11], or different (types of) population dynamics for each population [3,4,11,19].

Another line open to further investigation concerns the fact that in everyday conversations there is a strong sensitivity to face to face interaction affecting the strategies employed by the speakers in using one language instead of the other one, for example when an individual interacts with another individual whose linguistic identity is not certain. Despite most sociolinguistic surveys take these variables into account, on the level of data processing and mathematical modeling the situation is different and this aspect is rarely studied. This is best done in the framework of game-theoretical models, see e.g., [10,18].

Acknowledgments

This work has been supported by the targeted financing project SF0690030s09 and the Estonian Science Foundation through Grant No. 9462 (EH, MP).

References

[1] Abrams, D. M. and Strogatz, S. H., Modelling the dynamics of language death, *Nature* **424** (2003) 900.

[2] Aracil, L., El bilinguisme com a mite, in *Papers de Sociolinguistica* (1982), pp. 39–57 (in Catalan).

[3] Baggs, I. and Freedman, H., A mathematical model for the dynamical interactions between a unilingual and bilingual population: Persistence versus extinction, *J. Math. Sociol.* **16** (1990) 51.

[4] Baggs, I. and Freedman, H., Can the speakers of a dominated language survive as unilinguals — a mathematical-model of bilingualism, *Math. Comput. Model.* **18** (1993) 9.

[5] Castelló, X., Baronchelli, A. and Loreto, V., Consensus and ordering in language dynamics, *Eur. Phys. J. B* **71** (2009) 557.

[6] Castelló, X., Eguíluz, V. M. and Miguel, M. S., Ordering dynamics with two non-excluding options: Bilingualism in language competition, *New J. Phys.* **8** (2006) 306.

[7] Chapel, L., Castelló, X., Bernard, C., Deffuant, G., Eguíluz, V., Martin, S. and San Miguel, M., Viability and resilience of languages in competition, *PLoS ONE* **5** (2010) e8681.

[8] Palau. A. D. (ed.), *Una radiografia social de la llengua catalana*, Generalitat de Catalunya, Departament de Cultura, Cossetánia (2002) (in Catalan).

[9] Contento, S. (ed.), *Cresecere nel bilinguismo. Aspetti cognitivi, linguistici ed emotivi* (Carocci, Roma, 2010) (in Italian).
[10] Iriberri, N. and Uriarte, J., Minority language and the stability of bilingual equilibria, *Rationality Society* **24** (2012) 442.
[11] Kandler, A., Demography and language competition, *Hum. Biol.* **81** (2009) 181.
[12] Minett, J. and Wang, W.-Y., Modelling endangered languages: The effects of bilingualism and social structure, *Lingua* **118** (2008) 19.
[13] Mira, J. and Paredes, A., Interlinguistic similarity and language death dynamics, *Europhys. Lett.* **69** (2005) 1031.
[14] Mira, J., Seoane, L. and Nieto, J., The importance of interlinguistic similarity and stable bilingualism when two languages compete, *New J. Phys.* **13** (2011) 033007.
[15] Otero-Espinar, M., Seoane, L., Nieto, J. and Mira, J., An analytic solution of a model of language competition with bilingualism and interlinguistic similarity, *Physica D* **264** (2013) 17.
[16] Patriarca, M. and Heinsalu, E., Influence of geography on language competition, *Physica A* **388** (2009) 174.
[17] Patriarca, M. and Leppänen, T., Modeling language competition, *Physica A* **338** (2004) 296.
[18] Patriarca, M., Castelló, X., Uriarte, J., Eguíluz, V. and Miguel, M. S., Modeling two-language competition dynamics, *Adv. Complex Syst.* **15** (2012) 1250048.
[19] Pinasco, J. and Romanelli, L., Coexistence of languages is possible, *Physica A* **361** (2006) 355.
[20] Stauffer, D., Castelló, X., Eguíluz, V. M. and Miguel, M. S., Microscopic Abrams–Strogatz model of language competition, *Physica A* **374** (2007) 835.
[21] Vazquez, F., Castelló, X. and Miguel, M. S., Agent based models of language competition: Macroscopic descriptions and order-disorder transitions, *J. Stat. Mech.* (2010) P04007.
[22] Wichmann, S., The emerging field of language dynamics, *Lang. Linguist. Compass* **2/3** (2008) 442.
[23] Wichmann, S., Teaching & learning guide for: The emerging field of language dynamics, *Lang. Linguist. Compass* **2** (2008) 1294–1297.